B.sc(H) part 1 paper 1

Topic:solution of simultaneous equations of three variables

subject mathematics

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Inverse method

Ex 1

. **Solve the equations by matrix method,**

$$x - 2y + 3z = 5$$
$$4x + 3y + 4z = 7$$
$$x + y - z = -4.$$

Soln. Here $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 3 & 4 \\ 1 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & -2 & 3 \\ 4 & 3 & 4 \\ 1 & 1 & -1 \end{vmatrix}$

$= 1(-3-4) + 4(3-2) + 1(-8-9)$
$= -7 + 4 - 17 = -20 \neq 0.$

$\therefore$ A is non-singular.

The next step consists in finding out the inversion of A.

Consider matrix B_1 whose elements are the cofactors of the corresponding elements of $|A|$. It is

$$\begin{bmatrix} -7 & 8 & 1 \\ 1 & -4 & -3 \\ -17 & 8 & 11 \end{bmatrix}.$$

The transpose of $B_1 = \begin{bmatrix} -7 & 1 & -17 \\ 8 & -4 & 8 \\ 1 & -3 & 11 \end{bmatrix}$ which is = adj A.

$$\therefore \quad A^{-1} = \frac{1}{|A|}\begin{bmatrix} -7 & 1 & -17 \\ 8 & -4 & 8 \\ 1 & -3 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 7/20 & -1/20 & 17/20 \\ -2/5 & 1/5 & -2/5 \\ -1/20 & 3/20 & -11/20 \end{bmatrix}$$

Now $X = A^{-1}B$

$$\text{or,} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7/20 & -1/20 & 17/20 \\ -2/5 & 1/5 & -2/5 \\ -1/20 & 3/20 & -11/20 \end{bmatrix} \times \begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{4} - \frac{7}{20} - \frac{17}{5} \\ -2 + \frac{7}{5} + \frac{8}{5} \\ -\frac{1}{4} + \frac{21}{20} + \frac{11}{5} \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$x = -2, y = 1, z = 3.$

Ex 2. Test for consistency and solve the equation

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5.$$

Soln. The given system can be written in the matrix form

$$AX = B \text{ i.e., } \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}.$$

$$\text{where } A = \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}, \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ B = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}.$$

$$\text{Now, } |A| = \begin{vmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{vmatrix}$$

$$= 5(260 - 4) - 3(30 - 14) + 7(6 - 26 \times 7)$$

$$= 1280 - 48 - 1232 = 0.$$

$\therefore$ A is singular and hence A^{-1} does not exist.

Hence the given system has no solution or infinite solution according as $(\text{adj } A)B \neq 0$ or $(\text{adj } A)B = 0$.

We now find adj. A.

Consider matrix B_1 whose elements are the cofactors of the corresponding elements of $|A|$. It is

$$B_1 = \begin{bmatrix} 260 - 4 & -(30 - 14) & 6 - 7 \times 26 \\ -(30 - 14) & 50 - 49 & -(10 - 21) \\ 6 - 26 \times 7 & -(10 - 21) & (26 \times 5 - 9) \end{bmatrix}$$

$$= \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\therefore \text{adj } A = B_1' = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\therefore \quad (\text{adj } A)B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1024 - 144 - 830 \\ -64 + 9 + 55 \\ -704 + 9 + 605 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0.$$

Hence the system is consistent and it has infinite solutions.

Assuming $z = k$, the first two equations reduce to

$$5x + 3y = 4 - 7k$$

$$3x + 26y = 9 - 2k$$

which can be written in matrix form as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix} \text{ or } CX = D$$

where $C = \begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $D = \begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$.

$$\therefore \quad |C| = \begin{vmatrix} 5 & 3 \\ 3 & 26 \end{vmatrix} = 5 \times 26 - 9 = 130 - 9 = 121 \ (\neq 0).$$

$\therefore$ C is non-singular and hence C^{-1} exists.

Now, the cofactor matrix C_1 of the elements of C is

$$\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix} \quad \therefore \ C_1' = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$$

so that $(\text{adj } C) = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$.

$$\therefore \quad C^{-1} = \frac{1}{|C|}(\text{adj } C) = \frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}.$$

Hence $X = C^{-1}D = \frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$

$$= \frac{1}{121}\begin{bmatrix} 26(4-7k) - 3(9-2k) \\ -3(4-7k) + 5(9-2k) \end{bmatrix}$$

$$= \frac{1}{121}\begin{bmatrix} 104 - 182k - 27 + 6k \\ -12 + 21k + 45 - 10k \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{121}\begin{bmatrix} -176k + 77 \\ 11k + 33 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -16k + 7 \\ k + 3 \end{bmatrix}$$

$$\therefore \quad x = \frac{1}{11}(-16k + 7) \text{ and } y = \frac{1}{11}(k + 3).$$

Hence the solution is $x = \frac{-16k + 7}{11}, y = \frac{k + 3}{11}, z = k$